Communication Numérique

DSP théorique des codes en ligne

Yoann Morel

http://xymaths.free.fr/index.php?subdir=Signal

Soit $x(t) = g(t) * a(t)$ un signal codé en ligne, avec $a(t) = \sum_k \alpha_k \, \delta(t - kT)$, et donc, $x(t) = \sum_k \alpha_k \, g(t - kT)$.

$$\boxed{DSP(x)(f) = DSP(g)(f) \times \gamma_a(f)}$$

où,

$$\boxed{\gamma_a(f) = \frac{\sigma_a^2}{T} + \frac{\overline{\alpha}^2}{T^2} \Pi_{1/T}(f) + \frac{2}{T} \sum_{n=1}^{\infty} \Gamma_\alpha(n) \, \cos(2\pi n f T)}$$

avec les caractéristiques statistiques de la suite (α_k) :

- Moyenne : $\overline{\alpha} = E(\alpha_k)$
- Variance : $\sigma_\alpha^2 = \text{Var}(\alpha_k) = E(\alpha_k^2) - \overline{\alpha}^2 = \overline{\alpha^2} - \overline{\alpha}^2$
- Auto-corrélation : $R_{\alpha\alpha}(n) = E(\alpha_k \, \alpha_{k+n}) = \Gamma_\alpha(n) + \overline{\alpha}^2$

DSP du code unipolaire NRZ

$$g(t) = V\,\text{Rect}_{T_b}(t) \quad \Longrightarrow \quad \underline{|\widehat{g}(f)|^2 = V^2T^2\text{sinc}^2(fT_b)}$$

$$\alpha_k \in \{0;1\}\,,\ \text{donc,}\ \begin{cases} \overline{\alpha} = \frac{1}{2} \\ \sigma_\alpha^2 = \overline{\alpha^2} - \overline{\alpha}^2 = \frac{1}{2} - \left(\frac{1}{2}\right)^2 = \frac{1}{4} \\ \Gamma_\alpha(k) = 0\,;\ \forall\, k \geq 1 \end{cases}$$

D'où,

$$\text{DSP}(f) = \frac{V^2T}{4}\text{sinc}^2(fT_b) + \frac{V^2}{4}\text{sinc}^2(fT_b)\,\text{Ш}_{1/T_b}(f)$$

Or, $\text{sinc}(fT_b) = 0 \Longleftrightarrow f = \frac{k}{T_b}, \forall k \in \mathbb{N}^*$

d'où, finalement,

$$\boxed{\text{DSP}(f) = \frac{V^2T}{4}\text{sinc}^2(fT_b) + \frac{V^2}{4}\delta(f)}$$

DSP du code polaire NRZ

$$g(t) = V\text{Rect}_{T_b}(t) \quad \Longrightarrow \quad \underline{|\widehat{g}(f)|^2 = V^2 T^2 \text{sinc}^2(fT_b)}$$

$$\alpha_k \in \{-1;1\}\,,\ \text{donc,}\ \begin{cases} \overline{\alpha} = 0 \\ \sigma_\alpha^2 = \overline{\alpha^2} - \overline{\alpha}^2 = 1 - 0 = 1 \\ \Gamma_\alpha(k) = 0\,;\ \forall\, k \geq 1 \end{cases}$$

d'où,

$$\boxed{\text{DSP}(f) = V^2 T \text{sinc}^2(fT_b)}$$

DSP du code unipolaire RZ

$$g(t) = V\,\text{Rect}_{T_b/2}(t) \quad \Longrightarrow \quad \underline{|\widehat{g}(f)|^2 = \frac{V^2T^2}{4}\text{sinc}^2\left(\frac{fT_b}{2}\right)}$$

$$\alpha_k \in \{0;1\}\,,\ \text{donc,}\ \begin{cases} \overline{\alpha} = \frac{1}{2} \\ \sigma_\alpha^2 = \overline{\alpha^2} - \overline{\alpha}^2 = \frac{1}{2} - \left(\frac{1}{2}\right)^2 = \frac{1}{4} \\ \Gamma_\alpha(k) = 0\,;\ \forall\, k \geq 1 \end{cases}$$

D'où,

$$\text{DSP}(f) = \frac{V^2T}{16}\text{sinc}^2\left(\frac{fT_b}{2}\right) + \frac{V^2}{16}\text{sinc}^2\left(\frac{fT_b}{2}\right)\,\Pi_{1/T_b}(f)$$

DSP du code unipolaire RZ

$$\text{Or, sinc}\left(\frac{fT_b}{2}\right) = \begin{cases} 1 & \text{si } f = 0 \\ 0 & \text{si } f = \dfrac{2k}{T_b}, \ \forall k \in \mathbb{N}^* \\ \dfrac{(-1)^k}{k\pi + 1/2} & \text{si } f = \dfrac{2k+1}{T_b}, \ \forall k \in \mathbb{N}^* \end{cases}$$

d'où, finalement,

$$\text{DSP}(f) = \frac{V^2 T}{16}\text{sinc}^2\left(\frac{fT_b}{2}\right) + \frac{V^2}{16}\text{sinc}^2\left(\frac{fT_b}{2}\right)\Pi_{1/T_b}(f)$$

$$= \frac{V^2 T}{16}\text{sinc}^2\left(\frac{fT_b}{2}\right) + \frac{V^2}{16}\sum_{k\geq 1}\frac{1}{(k\pi + 1/2)^2}\delta\left(f - \frac{2k+1}{T_b}\right)$$

DSP du code polaire RZ

$$g(t) = V\,\mathsf{Rect}_{T_b/2}(t) \quad \Longrightarrow \quad \underline{|\widehat{g}(f)|^2 = \frac{V^2T^2}{4}\mathsf{sinc}^2\left(\frac{fT_b}{2}\right)}$$

$$\alpha_k \in \{-1;0;1\}\,,\ \text{donc,}\ \begin{cases} \overline{\alpha} = 0 \\ \sigma_\alpha^2 = \overline{\alpha^2} - \overline{\alpha}^2 = \frac{1}{2} \\ \Gamma_\alpha(1) = -\frac{1}{4}\ ;\ \Gamma_\alpha(k) = 0\,;\ \forall\, k \geq 2 \end{cases}$$

d'où,

$$\begin{aligned} \mathsf{DSP}(f) &= \frac{V^2T}{8}\mathsf{sinc}^2\left(\frac{fT_b}{2}\right) - \frac{V^2T^2}{8}\mathsf{sinc}^2\left(\frac{fT_b}{2}\right)\cos(2\pi fT_b) \\ &= \frac{V^2T}{8}\mathsf{sinc}^2\left(\frac{fT_b}{2}\right)\left[1 - \cos(2\pi fT_b)\right] \end{aligned}$$

et finalement,

$$\boxed{\mathsf{DSP}(f) = \frac{V^2T}{8}\mathsf{sinc}^2\left(\frac{fT_b}{2}\right)\sin^2(\pi fT_b)}$$

DSP du code Manchester

$$g(t) = V\,\mathrm{Rect}_{T_b/4}(t + T/2) + V\,\mathrm{Rect}_{T_b/4}(t - T/2)$$

$$\Longrightarrow \quad |\widehat{g}(f)|^2 = V^2 T^2 \frac{\sin^2\left(\frac{\pi f T_b}{2}\right)}{\pi f T_b/2}$$

$$\alpha_k \in \{-1;1\}\,, \text{ donc, } \begin{cases} \overline{\alpha} = 0 \\ \sigma_\alpha^2 = \overline{\alpha^2} - \overline{\alpha}^2 = 1 \\ \Gamma_\alpha(1) = 0\,, \ \forall k \geq 1 \end{cases}$$

et finalement,

$$\boxed{\mathrm{DSP}(f) = V^2 T \frac{\sin^4\left(\frac{\pi f T_b}{2}\right)}{(\pi f T_b/2)^2}}$$

DSP du code AMI

$$g(t) = V\text{Rect}_{T_b}(t) \quad \Longrightarrow \quad \underline{|\widehat{g}(f)|^2 = V^2T^2\text{sinc}^2(fT_b)}$$

$$\alpha_k \in \{-1; 0; 1\}, \text{ donc, } \begin{cases} \overline{\alpha} = 0 \\ \sigma_\alpha^2 = \overline{\alpha^2} - \overline{\alpha}^2 = \frac{1}{2} \\ \Gamma_\alpha(1) = -\frac{1}{4}\,; \Gamma_\alpha(k) = 0 \ \forall\, k \geq 2 \end{cases}$$

D'où,

$$\begin{aligned} \text{DSP}(f) &= \frac{V^2T}{2}\text{sinc}^2(fT_b) - \frac{V^2T}{2}\text{sinc}^2(fT_b)\cos(\pi fT_b) \\ &= \frac{V^2T}{2}\text{sinc}^2(fT_b)\left[1 - \cos(\pi fT_b)\right] \end{aligned}$$

d'où, finalement,

$$\boxed{\text{DSP}(f) = V^2T\frac{\sin^4(\pi fT_b)}{(\pi fT_b)^2)}}$$